Injecting Onto Off-Momentum Orbits in the Main Ring or the Main Injector without Changing the Booster

and Application to Slip Stacking

John Marriner
February 14, 1997

An orbit in a circular accelerator can be characterized by the beam velocity (v) and the orbit length (L). The rf frequency (*f*) corresponding to the orbit is:

$$f = h\frac{v}{L} \qquad [1]$$

where *h* is the harmonic number of the rf. If we transfer beams synchronously between the two machines, when the rf frequencies are locked to each other and the beam velocities of the orbits are equal, then we guarantee that the orbit lengths are in the ratio of the harmonic numbers of the two machines. Under these circumstances it is possible to make a bucket-to-bucket transfer between the two machines.

The formula for frequency [1] applies to all orbits, off-momentum orbits as well as the central orbit. The beam velocity is a function of the momentum (for a fixed particle mass) and the orbit length is a function of the magnetic field[1] and the momentum. We can write the relationship as

$$f = h\frac{v(p)}{L(B,p)} \qquad [2]$$

Small deviations from the nominal values are found by differentiation

$$\Delta f = f_0\left[\frac{1}{v_0}\frac{dv}{dp}\Delta p - \frac{1}{L_0}\left(\frac{\partial L}{\partial p}\right)_B \Delta p - \frac{1}{L_0}\left(\frac{\partial L}{\partial B}\right)_p \Delta B\right] \qquad [3]$$

where 0's refer to the nominal values and the Δ's to deviations from the nominal values. The derivatives are well known and [3] can be rewritten as:

$$\Delta f = f_0\left[\frac{1}{\gamma^2}\frac{\Delta p}{p_0} - \frac{1}{\gamma_t^2}\frac{\Delta p}{p_0} + \frac{1}{\gamma_{t\ \ p}^2}\frac{\Delta B}{B_0}\right] \qquad [4]$$

We assume that the matching conditions hold for the nominal values so that

$$p_m = p_b = p_0 \qquad [5]$$

---

[1] In principle, *all* the magnetic fields should be scaled by the same amount to leave the machine properties unchanged. However, for the small changes discussed here the most important effect comes from the main bending magnets.

$$f_m = f_b = f_0 \qquad [6]$$

$$\frac{h_b}{L_b} = \frac{h_m}{L_m} \qquad [7]$$

Note that any two of [5]-[7] imply the third. We wish to inject into the Main Ring or Main Injector with a frequency offset Δf_s, where Δf_s is the frequency difference between the central orbit and the off-momentum orbit. The total frequency shift Δf is related to Δf_s by

$$\Delta f = \Delta f_s + \Delta f_c \qquad [8]$$

where Δf_c is the frequency change of the central orbit arising from the change in magnetic field. The orbit length is fixed in this case and the momentum change is proportional to the change in magnetic field so it follows from [4] that

$$\frac{\Delta f_c}{f_0} = \frac{1}{\gamma^2}\frac{\Delta B}{B_0} \qquad [9]$$

where $\Delta B/B_0$ refers to the Main Ring (the Booster is zero by assumption). Matching the Main Ring and Booster requires that the total momentum and frequency shifts be equal as described by the following equations:

$$\Delta f = \left(-\eta_m \frac{\Delta p}{p_0} + \frac{1}{\gamma_{tm}^2}\frac{\Delta B}{B_0}\right) f_0 \qquad [10]$$

$$\Delta f = -\eta_b \frac{\Delta p}{p_0} f_0 \qquad [11]$$

where $\eta_b = \left(1/\gamma_t^w\right)_b - 1/\gamma^2$ is the slip factor for the Booster. The slip factors for the Main Injector and Main Ring are similarly defined. The solution of [8]-[10] is

$$\frac{\Delta p}{p_0} - \frac{\Delta B}{B_0} = -\frac{1}{\eta_m}\frac{\Delta f_s}{f_0} \qquad [12]$$

$$\frac{\Delta B}{B_0} = -\gamma_{tb}^2\left[\frac{\Delta f_s}{f_0} + \eta_b\left(\frac{\Delta p}{p_0} - \frac{\Delta B}{B_0}\right)\right] \qquad [13]$$

We can calculate the radial offsets for the beam at high dispersion points in the Booster by

$$\Delta r_b = \alpha_{pb}\frac{\Delta p}{p_0} \qquad [14]$$

and in the Main Injector or Main Ring by taking the displacement from the central orbit

$$\Delta r_m = \alpha_{pm}\left(\frac{\Delta p}{p_0} - \frac{\Delta B}{B_0}\right) \qquad [14]$$

The injection scheme for slip staking is to inject the first batch with a frequency offset of Δf_s. Injection is accomplished by phase locking to the "A" low level rf signal which drives the "A" cavities and has been set to the correct frequency. The beam is decelerated to a frequency of $-\Delta f_s$ using the "A" rf system. The second batch is injected by phase locking to the "B" rf system which is also set to a frequency offset of Δf_s. However, the kicker timing for the transfer must use the markers from the "A" system. Numerical values for the frequency and momentum offsets are given below. A frequency offset Δf_s =630 Hz is chosen so that the two beams (with frequencies $\pm\,\Delta f_s$) slip by one batch per Booster cycle. This choice of frequency offset is important when slip stacking over the full circumference of the Main Injector, but is arbitrary when only two booster batches are being stacked. The nominal parameters are assumed to be:

$$p_0 = 8888.9 \text{ MeV/c}$$

$$f_0 = 52.881669 \text{ MHz} \qquad [15]$$

$$\gamma = 9.526$$

Table of numerical values for off-momentum injection for slip stacking.

	Booster w/ MI	Main Injector	Booster w/MR	Main Ring	Units
Δf_s	630	630	630	630	Hz
γ_t	5.4	21.6	5.4	18.7	
η	0.0233	-0.0089	0.0233	-0.0082	
h	84	588	84	1113	
$\Delta p/p$-$\Delta B/B$		1.344		1.462	$\times10^{-3}$
$\Delta B/B$		-1.260		-1.340	MeV/c
$\Delta p/p$	0.084	0.084	0.122	0.122	$\times10^{-3}$
Δp	0.75	0.75	1.08	1.08	MeV/c
Δf	-103	-103	-150	-150	Hz
Δf_c		-733		-780	Hz
L	474.2	3319.4	474.2	6283.2	m
ΔL	1.37	9.56	1.98	26.27	mm
$\Delta L/L$	2.88	2.88	4.18	4.18	$\times10^{-6}$
α_p *(max)*	3	1.9	3	6	m
Δr *(max)*	0.25	2.55	0.37	8.77	mm